

SMI-19-96

# Composite p-branes in diverse dimensions

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## Abstract

We use a simple algebraic method to find a special class of composite p-brane solutions of higher dimensional gravity coupled with matter. These solutions are composed of  $n$  constituent p-branes corresponding  $n$  independent harmonic functions. A simple algebraic criteria of existence of such solutions is presented. Relations with  $D = 11$ ,  $D = 10$  known solutions are discussed.

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# 1 Introduction

The recent successful microscopic interpretation of the Bekenstein-Hawking entropy within string theory [1] has stimulated an investigation of p-brane solutions [2]-[10] of the low energy field equations of the string theory. In view of suggestions [11] that D=11 supergravity may be a low-energy effective field theory of a fundamental "M-theory" which generalizes known string theories it is important to find all classical p-brane solutions. It is also interesting to understand what types of D=11 solutions may exist in various dimension space-time. Some p-brane solutions of supergravity theory may be associated with non-perturbative state in superstring theory, like monopoles or dyones in Yang-Mills theory. P-brane solutions of low-energy string action provide also an evidence for existence of string duality.

The simplest theories that are relevant to superstring and supergravity theories are described by actions

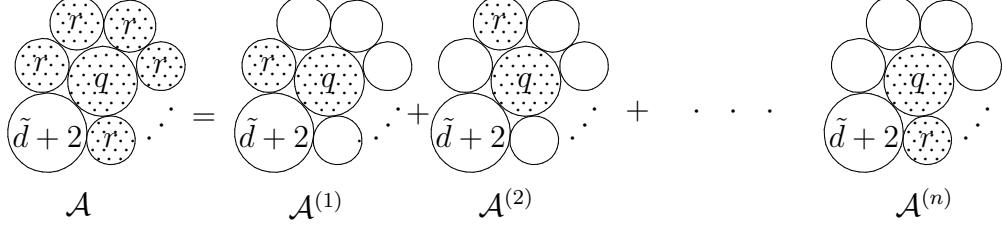
$$I = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} (R - \frac{1}{2(d+1)!} F_{d+1}^2), \quad (1.1)$$

and

$$I = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} (R - \frac{(\nabla\phi)^2}{2} - \frac{e^{-\alpha\phi}}{2(d+1)!} F_{d+1}^2), \quad (1.2)$$

where  $F_{d+1}$  is the  $d+1$  differential form,  $F_{d+1} = d\mathcal{A}_d$ ,  $\phi$  is a dilaton. In general, the fields  $F$  and  $\phi$  are linear combinations of the large variety of gauge fields and scalars.

The aim of this paper is to find solutions of equations of motions of the theories (1.1) and (1.2) that have so-called thin structure. In the p-brane terminology [12, 18, 20, 21, 22] these solutions correspond to  $n$  constituent p-branes. We will use an algebraic methods (cf.[8, 24, 25]) of finding solutions. The advantage of this simple method is its universality. The cases of different dimensions and different set of matter fields are treated in the same way. According to this method one can reduce the problem of finding solutions to an algebraic problem of solving an overdetermined non-linear system of algebraic equations. To make this reduction one uses algebraic requirements (see eqs.(2.11) below) which provide an absence of forces acting on matter sources and permit to reduce equations for matter fields to the harmonically conditions of functions describing matter fields. These conditions are direct analogues of "no-force" condition which has been recently used by Tseytlin [13] to get various BPS configurations in  $D = 11$  and  $D = 10$  cases (see also[2, 4, 7, 8]). These results are consistent with other approaches (D-brane supersymmetry analysis or study of potential between D-branes [14, 15, 16, 18, 20, 32, 17]). We will consider arbitrary dimension space-time and the case of "electric" solutions for one antisymmetric field. We will find an algebraic criteria of existence of solutions with thin structure. This criteria gives a restriction on dimension of space-time and parameters  $d$  and  $\alpha$ . For particular cases  $D = 10$  and  $D = 11$  this criteria is satisfied and some solutions reproduce the solutions recently found by Tseytlin [22] and Papadopoulos and Townsend [23] that after reductions produce supersymmetric BPS saturated p-brane solutions of low-dimensional theories [4, 6, 18]. A generalization to the case of magnetic solutions is straightforward, however a generalization for more fields with different dimensions need more treatment. This consideration is important in the context of [30]-[33].



## 2 Gravity + d-form

We will use the following ansatz for the metric

$$ds^2 = e^{2A} \eta_{\mu\nu} dy^\mu dy^\nu + \sum_{i=1}^n e^{2F_i} dz_i^{m_i} dz_i^{m_i} + e^{2B} dx^\gamma dx^\gamma, \quad (2.3)$$

$\mu, \nu$  run from 0 to  $q-1$ ,  $\eta_{\mu\nu}$  is a flat Minkowski metric,  $m_i, n_i$  run from 1 to  $r$ ,  $\alpha, \beta, \gamma$  run from 1 to  $\tilde{d}+2$ .  $A, B$  and  $C$  are functions on  $x$ . Under an assumption

$$qA + \sum r_i F_i + \tilde{d}B = 0, \quad (2.4)$$

the Ricci tensor for metric (2.3) has a simple form

$$R_{\mu\nu} = -h_{\mu\nu} e^{2(A-B)} \Delta A, \quad R_{m_i n_i} = -\delta_{m_i n_i} e^{2(F_i-B)} \Delta F_i \quad (2.5)$$

$$R_{\alpha\beta} = -q\partial_\alpha A \partial_\beta A - \sum_i r_i \partial_\alpha F_i \partial_\beta F_i + \tilde{d}\partial_\alpha B \partial_\beta B - \delta_{\alpha\beta} \Delta B \quad (2.6)$$

For matter field we use ansatz

$$\mathcal{A} = dy^0 \wedge dy^1 \wedge \dots \wedge dy^{q-1} \wedge [dz_1^1 \wedge \dots \wedge dz_1^{r_1} h_1 e^{C_1} + \dots + dz_n^1 \wedge \dots \wedge dz_n^{r_n} h_n e^{C_n}], \quad (2.7)$$

here  $r_i = r$ ,  $i = 1, \dots, n$ , i.e.  $D = q + nr + \tilde{d} + 2$ , and for  $r \neq 1$  the components of energy-momentum tensor read

$$T_{\mu\nu} = -\eta_{\mu\nu} e^{2(A-B)} \sum_{i=1}^n \frac{h_i^2}{4} e^{-2qA-2rF_i-2C_i} (\partial C_i)^2 \quad (2.8)$$

$$T_{m_i n_i} = \delta_{m_i n_i} e^{2(F_i-B)} \left[ -\frac{h_i^2}{4} e^{-2qA-2rF_i+2C_i} (\partial C_i)^2 + \sum_{j \neq i} \frac{h_j^2}{4} e^{-2qA-2rF_j+2C_j} (\partial C_j)^2 \right] \quad (2.9)$$

$$T_{\alpha\beta} = -\sum_{i=1}^n \frac{h_i^2}{2} e^{-2qA-2rF_i+2C_i} [\partial_\alpha C_i \partial_\beta C_i - \frac{\delta_{\alpha\beta}}{2} (\partial C_i)^2] \quad (2.10)$$

If we assume that

$$qA + rF_i = C_i, \quad i = 1, \dots, n \quad (2.11)$$

the form of  $T_{MN}$  crucially simplifies and the Einstein equations have the form

$$\Delta A = \sum_i t h_i^2 (\partial C_i)^2, \quad (2.12)$$

$$\Delta F_i = t h_i^2 (\partial C_i)^2 - \sum_{j \neq i} u h_j^2 (\partial C_j)^2 \quad (2.13)$$

$$\begin{aligned}
& -q\partial_\alpha A\partial_\beta A - \sum_i r_i \partial_\alpha F_i \partial_\beta F_i - \tilde{d}\partial_\alpha B\partial_\beta B - \delta_{\alpha\beta}\Delta B = \\
& - \sum_{i=1}^n \left[ \frac{h_i^2}{2} \partial_\alpha C_i \partial_\beta C_i - u \delta_{\alpha\beta} \right] (\partial C_i)^2
\end{aligned} \tag{2.14}$$

where  $t$  and  $u$  are given by

$$t = \frac{1}{2} \cdot \frac{D-2-q-r}{D-2}, \quad u = \frac{1}{2} \cdot \frac{q+r}{D-2} \tag{2.15}$$

The equation of motion for the antisymmetric field,

$$\partial_M (\sqrt{-g} F^{MM_1\dots M_d}) = 0, \tag{2.16}$$

under conditions (2.4) and (2.11) for the ansatz (2.7) reduces to

$$\partial_\alpha (e^{-C_i} \partial_\alpha C_i) = 0, \text{ or } \Delta C = (\partial C)^2. \tag{2.17}$$

The form of equation (2.17) shows a physical meaning of conditions (2.11). Under these conditions equations of motion for the antisymmetric field reduce to the harmonicity conditions for functions  $\exp(-C_i)$  describing matter field, i.e. there are no sources for  $\exp(-C_i)$ . These conditions correspond to "no-force" condition of vanishing of static force on a q-brane probe in gravitational background produced by another p-brane [2, 4, 7, 8, 13]. The LHS of (2.11) is the logarithm of the "Nambu" term for  $q+r$ -brane probe in the gravitational background (2.3) and the RHS of (2.11) is the logarithm of the "Wess-Zumino" term.

We solve equations (2.12) and (2.13) assuming

$$A = t \sum_i h_i^2 C_i, \tag{2.18}$$

$$F_i = th_i^2 C_i - u \sum_{j \neq i} h_j^2 C_j \tag{2.19}$$

To cancel the terms in front of  $\delta_{\alpha\beta}$  in equation (2.14) we also assume

$$B = -u \sum_i h_i^2 C_i \tag{2.20}$$

Note that if we want to solve equations for arbitrary harmonic functions  $H_i = \exp(-C_i)$  we have also to assume the conditions which follow from (2.11), (2.4) and nondiagonal part of (2.14). Equations (2.11) get the relations

$$(q+r)th_i^2 C_i + (qt - ru) \sum_{j \neq i} h_j^2 C_j = C_i, \quad i = 1, \dots, n \tag{2.21}$$

which under the assumption of independence of  $C_i$  give

$$qt - ru = 0, \tag{2.22}$$

$$(q+r)th_i^2 = 1. \tag{2.23}$$

Since  $t$  and  $u$  are given by the formulae (2.15) the condition (2.22) makes a restriction on dimensions  $D$ ,  $q$  and  $r$

$$q(D-2) = (q+r)^2 \quad (2.24)$$

Note that under this assumption the formula (2.15) has the form

$$t = \frac{1}{2} \frac{r}{q+r}, \quad u = \frac{1}{2} \frac{q}{q+r} \quad (2.25)$$

Equation (2.23) gives

$$h_i^2 = h^2 \equiv \frac{1}{(q+r)t}, \quad (2.26)$$

Equation (2.4) gives

$$qt + r(t - (n-1)u) - \tilde{d}u = 0, \quad (2.27)$$

Since  $D = q + rn + \tilde{d} + 2$  and  $qt = ru$  equation (2.22) is equivalent to  $q(D-2) = (q+r)^2$ , that we have just assumed.

Equation (2.14) gives two types of relations (2.20). One type of relations follows from the requirement of compensation of terms  $\partial_\alpha C_i \partial_\beta C_i$  in the both sides of equation (2.14) and the second one produces a compensation of mixed terms  $\partial_\alpha C_i \partial_\beta C_j, i \neq j$ . Straightforward calculations show that the both type of terms compensate.

This calculation shows that the form of metric is

$$ds^2 = (H_1 H_2 \dots H_n)^{-4t/r} \eta_{\mu\nu} dy^\mu dy^\nu + (H_1 H_2 \dots H_n)^{4u/r} \left[ \sum_{i=1}^n H_i^{-2/r} dz_i^{m_i} dz_i^{m_i} + dx^\gamma dx^\gamma \right], \quad (2.28)$$

or due to (2.25)

$$ds^2 = (H_1 H_2 \dots H_n)^{2q/r(q+r)} \left[ (H_1 H_2 \dots H_n)^{-2/r} \eta_{\mu\nu} dy^\mu dy^\nu + (H_1)^{-2/r} dz_1^{m_1} dz_1^{m_1} + \dots + (H_n)^{-2/r} dz_n^{m_n} dz_n^{m_n} + dx^\gamma dx^\gamma \right], \quad (2.29)$$

In Section 4 we will present solutions of equation (2.24).

### 3 Gravity + dilaton + d-form

We use the same ansatz for the metric and for the antisymmetric field but now in addition to the relation (2.4) we suppose the following conditions

$$-\alpha\phi - 2q^{(\alpha)} A^{(\alpha)} - 2r^{(\alpha)} F_i^{(\alpha)} - 2C_i = 0, \quad i = 1, \dots, n \quad (3.30)$$

In this case the form of  $\mu\nu$  and  $m_i n_i$  components of the Einstein equations does not change

$$\Delta A^{(\alpha)} = \sum_i t^{(\alpha)} h_i^{(\alpha)2} (\partial C_i)^2, \quad (3.31)$$

$$\Delta F_i^{(\alpha)} = t^{(\alpha)} h_i^{(\alpha)2} (\partial C_i)^2 - \sum_{j \neq i} u^{(\alpha)} h_j^{(\alpha)2} (\partial C_j)^2 \quad (3.32)$$

and there is one extra term in the RHS of  $\alpha\beta$  equations

$$\begin{aligned} -q\partial_\alpha A\partial_\beta A - \sum_i r_i \partial_\alpha F_i \partial_\beta F_i - \tilde{d}\partial_\alpha B\partial_\beta B - \delta_{\alpha\beta} \Delta B = \\ - \sum_{i=1}^n \left[ \frac{h_i^{(\alpha)2}}{2} \partial_\alpha C_i \partial_\beta C_i - u \delta_{\alpha\beta} \right] (\partial C_i)^2 \end{aligned} \quad (3.33)$$

Now the parameters  $t^{(\alpha)}$  and  $u^{(\alpha)}$  are given by the formula

$$t^{(\alpha)} = \frac{1}{2} \left[ 1 - \frac{q^{(\alpha)} + r^{(\alpha)}}{D^{(\alpha)} - 2} \right], \quad u^{(\alpha)} = \frac{1}{2} \frac{q^{(\alpha)} + r^{(\alpha)}}{D^{(\alpha)} - 2} \quad (3.34)$$

Equations of motion (2.16) under conditions (2.4) and (3.30) for the ansatz (2.7) reduce as in the previous case to the harmonicity conditions (3.35) for functions  $\exp(-C_i)$ . Equation of motion for dilaton reads

$$\Delta\phi - \frac{\alpha}{2} \sum_i h_i^{(\alpha)2} (\partial C_i)^2 \quad (3.35)$$

We solve equation (3.35) assuming

$$\phi = \frac{\alpha}{2} \sum_i h_i^{(\alpha)2} C_i, \quad (3.36)$$

and we solve equations (3.31) and (3.32) the relation (3.36) (now we put indexes  $\alpha$  for coefficients).

To cancel the terms in front of  $\delta_{\alpha\beta}$  in equation (3.33) we also assume

$$B^{(\alpha)} = -u^{(\alpha)} \sum_i h_i^{(\alpha)2} C_i \quad (3.37)$$

Note that if we want to solve equations for arbitrary harmonic functions  $H_i = \exp(-C_i)$  we have also to assume the conditions which follow from (3.30), (2.4) and nondiagonal part of (3.33). Equations (3.30) get the relations

$$\left[ \frac{\alpha^2}{4} + (q^{(\alpha)} + r^{(\alpha)}) t^{(\alpha)} \right] h_i^{(\alpha)2} C_i + \left[ \frac{\alpha^2}{4} + q^{(\alpha)} t^{(\alpha)} - r^{(\alpha)} u^{(\alpha)} \right] \sum_{j \neq i} h_j^{(\alpha)2} C_j = C_i, \quad i = 1, \dots, n \quad (3.38)$$

which under the assumption of independence of  $C_i$  give

$$\frac{\alpha^2}{4} = r^{(\alpha)} u^{(\alpha)} - q^{(\alpha)} t^{(\alpha)}, \quad (3.39)$$

$$\left[ \frac{\alpha^2}{4} + (q^{(\alpha)} + r^{(\alpha)}) t^{(\alpha)} \right] h_i^{(\alpha)2} = 1. \quad (3.40)$$

Since  $t^{(\alpha)}$  and  $u^{(\alpha)}$  are given by the formulae (3.34) the condition (3.39) makes a restriction on dimensions  $D^{(\alpha)}$ ,  $q^{(\alpha)}$  and  $r^{(\alpha)}$

$$\left( \frac{\alpha^2}{2} + q^{(\alpha)} \right) (D^{(\alpha)} - 2) = (q^{(\alpha)} + r^{(\alpha)})^2 \quad (3.41)$$

Note that under this assumption the formulae (3.34) have the form

$$u^{(\alpha)} = \frac{1}{4} \frac{2q^{(\alpha)} + \alpha^2}{q^{(\alpha)} + r^{(\alpha)}}, \quad t^{(\alpha)} = \frac{1}{4} \frac{2r^{(\alpha)} - \alpha^2}{q^{(\alpha)} + r^{(\alpha)}} \quad (3.42)$$

The LHS of (3.40) for  $t^{(\alpha)}$  and  $u^{(\alpha)}$  given by formula (3.42) can be represented as  $r^{(\alpha)}h^{(\alpha)2}/2$ , and, therefore, equation (3.40) gives

$$h_i^{(\alpha)2} = h^{(\alpha)2} \equiv \frac{2}{r^{(\alpha)}}, \quad (3.43)$$

As in the case of absence of dilaton one can check that for  $\alpha$ ,  $q^{(\alpha)}$ ,  $r^{(\alpha)}$  and  $D^{(\alpha)}$  satisfying relation (3.41) and  $h^{(\alpha)}$  given by (3.43) equations (2.4) as well as equations given a compensation of terms  $\partial_\alpha C_j \partial_\beta C_i$  ( $i = j$  as well as  $i \neq j$ ) in the both sides of equation (3.33) are fulfilled.

This calculation shows that the metric

$$\begin{aligned} ds^2 = & (H_1 H_2 \dots H_n)^{-4t^{(\alpha)}/r^{(\alpha)}} \eta_{\mu\nu} dy^\mu dy^\nu + \\ & (H_1 H_2 \dots H_n)^{4u^{(\alpha)}/r^{(\alpha)}} \left[ \sum_{i=1}^n H_i^{-2/r^{(\alpha)}} dz_i^{m_i} dz_i^{m_i} + dx^\gamma dx^\gamma \right] = \\ & (H_1 H_2 \dots H_n)^{4u^{(\alpha)}/r^{(\alpha)}} \left[ (H_1 H_2 \dots H_n)^{-2/r^{(\alpha)}} \eta_{\mu\nu} dy^\mu dy^\nu + \right. \\ & \left. \sum_{i=1}^n H_i^{-2/r^{(\alpha)}} dz_i^{m_i} dz_i^{m_i} + dx^\gamma dx^\gamma \right], \end{aligned} \quad (3.44)$$

and matter fields in the form

$$\exp \phi = (H_1 H_2 \dots H_n)^{-\alpha/r^{(\alpha)}} \quad (3.45)$$

$$\mathcal{A} = h^{(\alpha)} dy^0 \wedge dy^1 \wedge \dots \wedge dy^{q-1} \wedge [dz_1^1 \wedge \dots \wedge dz_1^r H_1^{-1} + \dots + dz_n^1 \wedge \dots \wedge dz_n^r H_n^{-1}] \quad (3.46)$$

are the solution of the theory.

## 4 Examples

Note that equation (2.24) is very restrictive since it has to be solved for integers. Let us present some examples.

For dimensions  $D = 4, 5, 6, 7, 8$  and  $9$  there are solutions only with  $r = 0$  and we have 2-block solutions with  $\tilde{d} = 0$ . In these cases, either the spacetime is asymptotically  $M^q \times Y$ , where  $Y$  is a two-dimensional conical space, or the metric exhibits logarithmic behavior as  $|x| \rightarrow \infty$  [26]-[27].

We get more interesting structures in  $D = 6$  case. There are four types of solutions.

- i)  $q = 1, r = 1, n = 2, \tilde{d} = 1$ ,
- ii)  $q = 1, r = 1, n = 3, \tilde{d} = 0$ ,
- iii)  $q = 1, r = 1, n = 4, \tilde{d} = -1$ ,
- iv)  $q = 1, r = 1, n = 5, \tilde{d} = -2$ .

Here we have to assume that different branches with  $r = 1$  correspond to different gauge field

$$\mathcal{A}^{(I)} = h dy^0 \wedge dz_i H_i^{-1} \delta_{iI},$$

I=1,...n.(Otherwise we cannot guarantee the diagonal form of the stress-energy tensor). The solution iv) is identified with the Minkowski vacuum of the theory. The solution iii) separates the 6-dimensional space-time into two asymptotic regions like a domain wall [27]. The metric for the solution with  $\tilde{d} = 0$  has a logarithmic behavior,  $H = \sum_a \ln(\frac{Q_a}{|x-x_a|^2})$ .

For  $\tilde{d} = 1$  we have

$$ds^2 = (H_1 H_2) [-(H_1 H_2)^{-2} dy_0^2 + (H_1)^{-2} dz_1^2 + (H_2)^{-2} dz_2^2 + (dx_i)^2], i = 1, 2, 3 \quad (4.1)$$

$$\mathcal{A}^{(1)} = \sqrt{2} dy_0 \wedge dz_1 H_1^{-1},$$

$$\mathcal{A}^{(2)} = \sqrt{2} dy_0 \wedge dz_2 H_2^{-1},$$

$$H_1 = 1 + \sum_{a=1}^{l_1} \frac{Q_a^{(1)}}{|x - x_a^{(1)}|}; \quad H_2 = 1 + \sum_{b=1}^{l_2} \frac{Q_b^{(2)}}{|x - x_b^{(2)}|}, \quad |x - x_a| = (\sum_{i=1}^3 |x_i - x_{a_i}|^2)^{1/2}. \quad (4.2)$$

If  $l_1 = l_2 = l$  and  $x_a^{(1)} = x_a^{(2)} = x_a$  the metric (4.1) has horizons at the points  $x_a$ . The area (per unit of length in all p-brane directions) of the horizons  $x = x_a$  is

$$A_4 = 4\pi \sum_{a=1}^l Q_a^{(1)} Q_a^{(2)} \quad (4.3)$$

This confirms an observation [28] that extremal black holes have non vanishing event horizon in the presence of two or more charges (electric or magnetic).There are more solutions for several scalar fields [31]-[33].

For  $D = 10$  we have two solutions with  $\tilde{d} = 0$ .

- i)  $q = 8, r = 0, \tilde{d} = 0, n = 1;$
- ii)  $q = 2, r = 2, \tilde{d} = 0, n = 3.$

There is also solution with  $q = 2, r = 2, \tilde{d} = 2, n = 2,$

$$\begin{aligned} ds^2 &= (H_1 H_2)^{1/2} [-(H_1 H_2)^{-1} (-dy_0^2 + dy_1^2 + K du^2) \\ &\quad + H_1^{-1} (dz_1^2 + dz_2^2) + H_2^{-1} (dz_3^2 + dz_4^2) + \sum_{i=1}^4 (dx_i)^2], \\ H_1 &= 1 + \sum_a \frac{Q_a^{(1)}}{|x - x_a|^2}; \quad H_2 = 1 + \sum_b \frac{Q_b^{(2)}}{|x - x_b|^2}, \\ K &= \sum_a \frac{Q_a}{|x - x_a|^2}; \quad u = y_0 + y_1 \end{aligned} \quad (4.4)$$

The area of this horizon (per unit length in all p-brane directions) is

$$A_8 = \omega_3 \sum_a (Q_a^{(1)} Q_b^{(2)} Q_a)^{1/2} \quad (4.5)$$

where  $\omega_3 = 2\pi^2$  is the area of the unit 3-dimension sphere. In this case the different components of the same gauge field act as fields corresponding to different charges. For  $D = 11$  we have the following solutions with  $\tilde{d} > 1$ .

- i)  $q = 1, r = 2, \tilde{d} = 2, n = 3,$

$$ds^2 = (H_1 H_2 H_3)^{1/3} [-(H_1 H_2 H_3)^{-1} dy_0^2$$

$$+ H_1^{-1}(dz_1^2 + dz_2^2) + H_2^{-1}(dz_3^2 + dz_4^2) + H_3^{-1}(dz_5^2 + dz_6^2) + \sum_{i=1}^4 dx_i^2], \quad (4.6)$$

$$H_c = 1 + \sum_a \frac{Q_{ac}}{|x - x_a|^2}; \quad c = 1, 2, 3.$$

For  $H_3$  this solution reproduces a solution found in [13]. We get non-zero area of the horizon  $x = x_a$ .

$$A_9 = \omega_3 \sum_a (Q_a^{(1)} Q_a^{(2)} Q_a^{(3)})^{1/2} \quad (4.7)$$

ii)  $q = 4, r = 2, \tilde{d} = 1, n = 2$ ,

$$ds^2 = (H_1 H_2)^{2/3} [(H_1 H_2)^{-1} (-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + H_1^{-1}(dz_1^2 + dz_2^2) + H_2^{-1}(dz_3^2 + dz_4^2) + \sum_{i=1}^3 dx_i^2], \quad (4.8)$$

where  $H_1$  and  $H_2$  are given by (4.2). This solution has been recently found in [13]. The area of the horizon  $x = x_a = x_b$  is equal to zero.

For  $D = 12, 13, 14, 15, 16, 17$ , there are only  $\tilde{d} = 0$  solutions.

For  $D = 18$  and  $D = 20$  there are the following solutions with  $\tilde{d} > 1$ .

$$D = 18, \text{ i) } q = 1, r = 3, \tilde{d} = 3, n = 4, H_c = \sum \frac{Q_{ac}}{(x - x_{ac})^3}, c = 1, 2, 3, 4.$$

$$\text{ii) } q = 4, r = 4, \tilde{d} = 4, n = 2, H_c = \sum \frac{Q_{ai}}{(x - x_{ac})^4}, c = 1, 2$$

$$D = 20, \text{ q = 5, r = 4, } \tilde{d} = 1, n = 3, H_c = \sum \frac{Q_{ac}}{(x - x_{ac})^3}, c = 1, 2, 3$$

Let us present some examples of solution of equation (3.42). Note that this equation gives a "quantized" values for  $\alpha$ .

For  $D = 10$  we have  $q = 1, r = 3, \alpha = \pm\sqrt{2}, \tilde{d} = 1, n = 2$  and the corresponding metric has the form

$$ds^2 = (H_1 H_2)^{1/3} [-H_1 H_2)^{-2/3} dy_0^2 + H_1^{2/3} (dz_1^2 + dz_2^2 + dz_3^2) + H_2^{-2/3} (dz_4^2 + dz_5^2 + dz_6^2) + \sum_{i=1}^3 dx_i^2], \quad (4.9)$$

where  $H_1$  and  $H_2$  are given by (4.2). This solution correspond to IIA supergravity. The area of the horizon  $x = x_a = x_b$  is equal to zero.

For  $D = 11$  we have the following solutions: i)  $q = 1, r = 5, \alpha = \pm\sqrt{6}, \tilde{d} = 3, n = 1$ ; ii)  $q = 2, r = 4, \alpha = \pm 2, \tilde{d} = 3, n = 1$ , iii)  $q = 3, r = 3, \alpha = \pm\sqrt{2}, \tilde{d} = 3, n = 1$ .

In conclusion, we have found multi-block p-brane solutions for high dimensional gravity interacting with matter. We have assumed the "electric" ansatz for matter field and using "non-force" conditions for local fields (2.11) together with the harmonic gauge condition (2.4) to reduce the system of differential equations to overdetermined system of non-linear equations. The found solutions support a picture in which the extremal p-brane can be viewed as a composite of 'constituent' branes, each of the latter possessing a charge corresponding to one of the gauge fields or only to one of the components for the decomposition (Fig.1). It would be interesting to perform the similar calculations for "dyonic" ansatz as well as to consider reduction of found solutions to low dimensional cases a la Kaluza-Klein [23].

## 5 Acknowledgments

We are grateful to A.Tseytlin for useful comments. This work is supported by the RFFI grant 96-01-00608. We thank M.G.Ivanov for taking our attention to a special case of  $r_i = 1$ .

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